MISSOURI WESTERN STATE UNIVERSITY

COLLEGE OF LIBERAL ARTS AND SCIENCES

DEPARTMENT OF COMPUTER SCIENCE, MATHEMATICS, AND PHYSICS

COURSE NUMBER: MAT 332

COURSE NAME: Probability Theory

COURSE DESCRIPTION:

The study of discrete and continuous probability distributions.

PREREQUISITE:

Credit in MAT 287.

TEXT:

TECHNOLOGY:

Students will utilize probability distribution commands on graphing calculators (such as the TI-83/84, the recommended calculator) to study select random variables. Demonstrations using the statistical software Minitab are used to illustrate the effects of changing parameters for a distribution.

COURSE OBJECTIVES:

The major goal of this course is to provide students with knowledge and understanding of the mathematical foundations of probability and the ability to apply that knowledge to solve problems. In order to meet this major goal, students will learn how to:

1. Compute probabilities of discrete and continuous random variables.
   (MoStep Mathematics Competencies 1.1, 1.2, 1.6, 4.1, 4.2, 7.1)

2. Find the probability density and distribution functions for discrete and continuous random variables.
   (MoStep Mathematics Competencies 1.1, 1.2, 1.10, 1.12, 2.5, 4.2, 5.3, 5.5, 7.1, 7.2, 8.2)

3. Compute mathematical expectations of random variables and functions of random variables.
   (MoStep Mathematics Competencies 1.1, 1.2, 1.6, 2.5, 5.3, 5.5, 8.2)

4. Identify frequently encountered distributions by name, by probability density function, and/or description of problem situations to which those distributions apply.
(MoStep Mathematics Competencies 1.2, 1.6, 4.2, 4.3, 4.4, 5.3, 5.5, 7.2)

5. Find the moment-generating function of selected discrete and continuous random variables.
(MoStep Mathematics Competencies 1.1, 5.2, 8.2)

6. Compute probabilities related to multivariate distributions.
(MoStep Mathematics Competencies 1.1, 1.2, 1.6, 1.12, 5.5, 7.1, 7.2, 8.2)

STUDENT COMPETENCIES:

1. Apply methods of enumeration to find the number of outcomes in the sample spaces of selected experiments and events related to those sample spaces.

2. Compute probabilities of events using results of enumeration methods.

3. Compute probabilities using the probability density function.

4. When given a probability density function, find the distribution function.

5. When given a distribution function, find the probability density function.

6. Prove basic theorems of probability using the defining properties of a probability function.

7. Compute probabilities using general and special theorems of probability.

8. Identify situations to which the following distributions apply: Binomial; Negative binomial; Geometric; Hypergeometric; Poisson; Exponential; Gamma; Normal; Chi-square.

9. Compute conditional probabilities.

10. Compute probabilities of jointly distributed random variables.

11. Utilize the concept of random variable and its probability density function to model and analyze real-world phenomena.

12. Build mathematical models for real world situations such as those modeled by the Poisson process.

13. Distinguish between discrete and continuous random variables and apply the appropriate techniques to each situation.

14. Identify probability as one of the major branches of mathematics.

15. Demonstrate the interrelationship of the different branches of mathematics by applying techniques of calculus and algebra to problems in probability as appropriate.

16. Utilize the moment generating function to find the mean and variance of selected discrete and continuous random variables.

17. Utilize Venn diagrams to motivate statements and/or proofs of probability theorems.

COURSE OUTLINE:

I. Probability
   - A. Probability and Inference
   - B. A Review of Set Notation
   - C. A Probabilistic Model for an Experiment: The Discrete Case
   - D. Calculating the Probability of an Event: The Sample-Point Method
   - E. Tools for Counting Sample Points
   - F. Conditional Probability and the Independence of Events
   - G. Two Laws of Probability
   - H. Calculating the Probability of an Event: The Event-Composition Method
   - I. The Law of Total Probability and Bayes' Rule
   - J. Numerical Events and Random Variables
   - K. Random Sampling

II. Discrete Random Variables and Their Probability Distributions
   - A. The Probability Distribution for a Discrete Random Variable
   - B. The Expected Value of a Random Variable or Function of a Random Variable
   - C. The Binomial Probability Distribution
   - D. The Geometric Probability Distribution
   - E. The Negative Binomial Probability Distribution
   - F. The Hypergeometric Probability Distribution
   - G. The Poisson Probability Distribution
   - H. Moments and Moment-Generating Functions
   - I. Probability-Generating Functions
   - J. Tchebysheff's Theorem

III. Continuous Random Variables and Their Probability Distributions
   - A. The Probability Distribution for a Continuous Random Variable
   - B. Expected Values for Continuous Random Variables
   - C. The Uniform Probability Distribution
   - D. The Normal Probability Distribution
   - E. The Gamma Probability Distribution

F. The Beta Probability Distribution
G. Some General Comments
H. Other Expected Values
I. Tchebysheff's Theorem
J. Expectations of Discontinuous Functions and Mixed Probability Distributions

IV. Multivariate Probability Distributions
A. Bivariate and Multivariate Probability Distributions
B. Marginal and Conditional Probability Distributions
C. Independent Random Variables
D. The Expected Value of a Function of Random Variables
E. Special Theorems
F. The Covariance of Two Random Variables
G. The Expected Value and Variance of Linear Functions of Random Variables
H. The Multinomial Probability Distribution
I. The Bivariate Normal Distribution
J. Conditional Expectations

V. Functions of Random Variables
A. Finding the Probability Distribution of a Function of Random Variables
B. The Method of Distribution Functions
C. The Method of Transformations
D. The Method of Moment-Generating Functions
E. Multivariate Transformations Using Jacobians
F. Order Statistics